

Experimental verification of the opposite effect of fluid loading on the velocity of dilatational waves in thin plates and rods

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In a recent paper [J. Acoust. Soc. Am. **102**, 3478 (1997)], it was demonstrated by analytical means that radiation loading increases the velocity of dilatational waves in immersed thin plates, but decreases it in thin rods. The main goal of this paper is to verify experimentally the predicted opposite effect, which is particularly interesting because in almost every respect, the lowest-order dilatational modes of wave propagation in thin plates and rods are very similar. Experimental verification of the predicted small radiation-induced velocity change is rather difficult, partly because of the accompanying strong attenuation effect caused by radiation losses, and partly because of the presence of an additional velocity change caused by viscous drag even in low-viscosity fluids like water. In spite of these inherent difficulties, the presented experimental results provide unequivocal verification of the earlier theoretical predictions. © 1999 Acoustical Society of America. [S0001-4966(99)03005-2]

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INTRODUCTION

The problem of wave propagation in fluid-loaded plates was extensively studied by numerous investigators. Schoch,¹ Osborne and Hart,² Merkulov,³ Viktorov,⁴ Pitts *et al.*,⁵ and Selezov *et al.*⁶ studied the effect of water loading on plates under a variety of conditions. Their results indicated that most modes are strongly attenuated by leaking into the fluid, but their phase velocity is essentially unaffected. More recent studies by Nayfeh and Chimenti⁷ and Rokhlin *et al.*⁸ showed that in some special cases of significant importance in non-destructive characterization of composite laminates, when the density ratio between the solid and the fluid is relatively low, the change in the phase velocity of the fundamental (lowest-order) symmetric mode is not negligible and, in extreme cases, even the topology of the mode structure might change. Most of the theoretical works on wave propagation in loaded rods considered the more general case of clad rods consisting of an isotropic core and an arbitrary number of isotropic coatings. A comprehensive review of these studies was published by Thurtson.⁹

Longitudinal guided-wave propagation in thin plates and rods is of great practical importance. Such guided modes are commonly used to evaluate the material properties of thin foils and plates, metal wires, optical fibers, and reinforcement filaments used in epoxy, metal, and ceramic matrix composites. The feasibility of using such longitudinal guided waves to evaluate the interface properties between the fiber and the surrounding solid matrix was investigated by both theoretical and experimental means.^{10,11} Another interesting application is when an imbedded fiber is used to monitor the properties of the surrounding material during poly-

merization.¹² In this case, the fiber properties are known and the interface conditions are assumed to be perfect. Any change in the velocity or the attenuation of the guided mode in the fiber can be attributed to the surrounding material that changes from a viscous fluid to an elastic solid during the curing process to be monitored. Guided modes producing mainly tangential displacement can be readily used to measure fluid viscosity. From this point of view, torsional modes are the best,^{13,14} but they are more difficult to generate and detect than extensional modes.¹⁵ Nagy and Kent recently utilized the fundamental symmetric mode in thin wires and fibers to assess their Poisson's ratio through measuring the leaky attenuation of these modes in such structures.¹⁶

At low frequencies, only the fundamental symmetric longitudinal and antisymmetric flexural modes can propagate in thin plates. In rods, a dispersion-free torsional mode can also propagate in addition to the longitudinal and flexural modes. The fundamental longitudinal modes in thin plates and rods are physically very similar. Figure 1 shows the schematic diagram of a thin plate and a rod along with the common deformation pattern caused by the fundamental dilatational mode. At sufficiently low frequencies, the displacement along the propagation (x) direction dominates the vibration. In a thin plate, the Poisson effect causes a weaker transverse vibration perpendicular to the surface of the plate, which is less than the longitudinal displacement by a factor proportional to $\omega d \nu / (1 - \nu)$, where ω denotes the angular frequency, d is the thickness of the plate, and ν is the Poisson ratio. The fundamental axisymmetric mode propagating in a thin rod exhibits a similar behavior. The displacement along the axis of the rod dominates the vibration, while the

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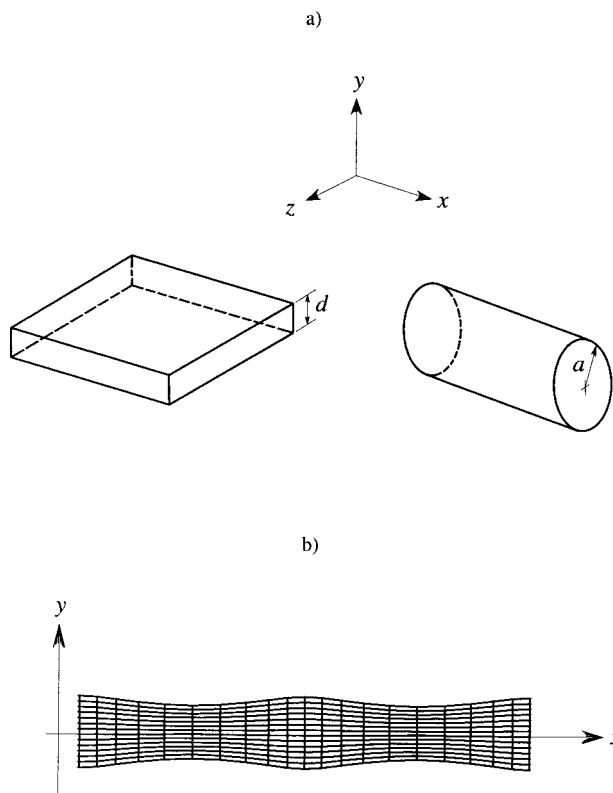


FIG. 1. The schematic diagram of a plate and a rod with the associated coordinate system (a) and the deformation pattern caused by the fundamental dilatational mode (b).

radial displacement caused by Poisson's effect is less than the longitudinal displacement by a factor proportional to $\omega a \nu$, where a is the radius of the rod. The only minor difference between these two modes is that in the case of a plate, the Poisson effect is restricted to only one dimension normal to the plate; therefore, the dilatational wave velocity is $\sqrt{1/(1-\nu^2)} \approx 1.05$ times higher in a plate than in a rod for a typical value of $\nu = 0.3$. As the frequency increases, in both cases the phase velocity first drops proportionally to the square of frequency and then asymptotically approaches the Rayleigh velocity. In spite of these common features of the dilatational vibrations in thin plates and rods, the effect of fluid loading on the phase velocity of these two modes is quite different.¹⁷ Although the magnitude of the relative change in the phase velocity due to radiation loading is very small and practically negligible compared to the much stronger attenuation effect in both cases, the very fact that the signs of these velocity changes are opposite deserves some attention. This fundamental difference in the effect of radiation loading on the velocity of the dilatational mode in thin plates and rods was thoroughly investigated by analytical means in Ref. 17. The phenomenon was first demonstrated by both numerical and asymptotic inspection of the well-known exact dispersion equations. Then, simplified approximate models were introduced to facilitate a better understanding of the physical differences between the two cases that lead to such an opposite behavior. It was found that the somewhat unexpected opposite effect of fluid loading on immersed rods and plates is caused by the different nature of their radiation loading. In the case of the plate, the radiation

impedance exerted by the fluid is purely real, whereas it is dominantly imaginary in the case of thin rods. Our main goal in this paper is to verify experimentally these theoretical predictions by measuring the relative change in the velocity of the dilatational waves in thin plates and rods immersed in water.

I. THEORETICAL CONSIDERATIONS

Generally, guided-wave propagation in plates and rods immersed in ordinary low-viscosity fluids like water is mainly affected by radiation loading due to the normal component of the surface vibration. The fundamental dilatational modes in thin plates and rods are somewhat different in that, due to the dominantly longitudinal vibration in the solid, the tangential component of the surface vibration can cause significant viscous drag. In this case, the total fluid-loading effect can be separated into comparable radiation and viscous parts. First, the normal component of the surface vibration causes radiation losses via energy leakage into the fluid, and slightly increases or decreases the velocity depending on whether we consider a plate or a rod. Second, the tangential component of the surface vibration causes additional losses via viscous dissipation in the fluid and slightly decreases the velocity in both plates and rods. Since the change in the velocity of this mode in thin plates and rods due to leakage into the fluid is very small, on the order of a fraction of a percent,¹⁷ careful considerations must be taken of all extraneous effects to successfully measure it. Detailed analysis of the various parameters affecting this phenomenon is needed to establish the physical limits imposed on such measurements and to design the optimal experiment. It is also necessary to pay proper attention to other usually negligible effects such as inherent temperature variations caused by immersion in the fluid bath, remnant wetness, and evaporation cooling when the specimen is removed from the bath, etc. In the following discussion of the various factors affecting our ability to successfully measure the sought radiation-induced velocity change in immersed plates and rods, we separate the relevant parameters into two categories. The primary factors include the frequency, the geometrical dimensions, and the acoustical properties of the solid and the fluid used in the experiment. The secondary factors are those due to viscosity and temperature variations. In the following sections we proceed to examine these different factors, their contributions, and their importance. The results of the analysis are then implemented in the design of the experiments that are aimed at measuring the effect of radiation-induced fluid loading on the velocity of the dilatational mode in thin plates and rods.

A. Radiation loading

In our experiment, the change in the velocity of the dilatational wave in thin plates and rods is determined at a given frequency by measuring the phase change in the received ultrasonic signal between the free case and the fluid-loaded case. To examine the effect of frequency, the dimension of the specimen, and the acoustical properties of the solid and the fluid on the accuracy of the measurement, and to establish their optimal values, we need to maximize the

radiation-induced change in the phase angle of the received signal against the attenuation of the signal. Too much attenuation will cause a serious degradation of the received ultrasonic signal and would result in significant errors in the phase measurement. On the other hand, too little attenuation is inherently accompanied by only a slight change in the phase of the received signal making it virtually impossible to measure the accurate phase variation. Towards this end, we define a “quality” factor Q as the ratio between the change in the phase angle $\Delta\Phi$ and the total loss of the signal ΔL over the fluid-loaded portion of the structure. The change in the phase angle can be approximated as

$$\Delta\Phi \approx \frac{\Delta c}{c_o^2} \omega /, \quad (1)$$

where Δc is the change in the velocity of the dilatational mode due to fluid loading, c_o is the low-frequency asymptotic limit of the velocity of the dilatational mode in the free structure, ω is the angular frequency, and $/$ is the length of the fluid-loaded portion of the structure. For the plate, $c_o = \sqrt{E/[\rho_s(1-\nu^2)]} = c_s \sqrt{2/(1-\nu)}$, and for the rod it is given as $c_o = \sqrt{E/\rho_s} = c_s \sqrt{2(1+\nu)}$, where E denotes Young’s modulus, ν is the Poisson ratio, c_s is the shear velocity in the solid, and ρ_s is its density. The total attenuation of the mode due to energy leakage into the fluid is calculated as $\Delta L = \alpha /$, where α is the attenuation coefficient of the leaky mode. Combining the above equations, we can express the quality factor as

$$Q = \frac{\omega \Delta c}{\alpha c_o^2}. \quad (2)$$

The low-frequency, weak-loading asymptotic expressions for the relative change in the velocity of the dilatational wave in thin plates and rods can be found in Ref. 17 and are repeated here for convenience. For the case of the plate, the relative velocity change due to radiation loading is given as

$$\left. \frac{\Delta c}{c_o} \right|_{\text{plate}} \approx \frac{1}{32} \left[\frac{\rho d \omega \nu c_f^2}{c_s^2} \right]^2, \quad (3)$$

whereas for the rod it can be expressed as

$$\left. \frac{\Delta c}{c_o} \right|_{\text{rod}} \approx \frac{\rho a^2 \omega^2 \nu^2}{2 c_s^2 (1+\nu)} \ln \left(\frac{a \omega}{c_o} \sqrt{\frac{c_o^2}{c_f^2} - 1} \right). \quad (4)$$

Here, $\rho = \rho_f / \rho_s$, is the density ratio between the fluid and the solid, d is the thickness of the plate, a is the radius of the rod, and c_f is the sound velocity in the fluid. The opposite sign of the radiation-induced velocity change in plates and rods is quite obvious from Eqs. (3) and (4). The quantity given for a plate in Eq. (3) is always positive, while the corresponding quantity given for a rod in Eq. (4) is negative for low values of the normalized frequency $a \omega / c_o$. The low-frequency asymptotic expressions for the leaky attenuation coefficient of this mode are listed in Ref. 16 and are also repeated here for convenience. For the plate, we have

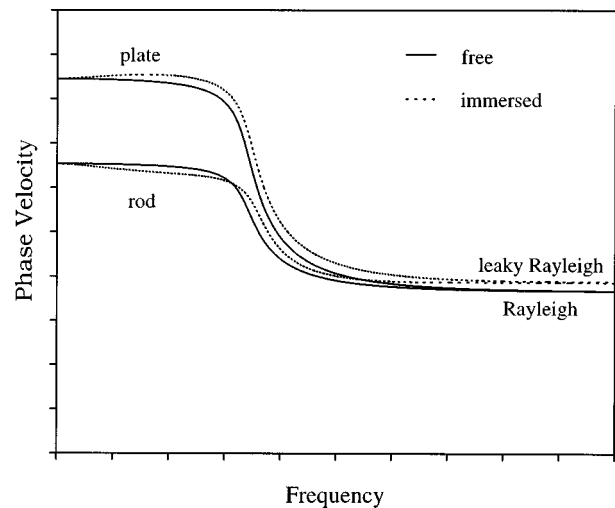


FIG. 2. Schematic diagram showing the fluid-loading induced changes in the topology of the dispersion curves for the fundamental dilatational modes in thin plates and rods (the changes in the phase velocity have been exaggerated for clarity).

$$\alpha_{\text{plate}} \approx \frac{\rho d \omega^2 \nu^2}{4 c_o^2 (1-\nu)^2 \sqrt{\frac{c_o^2}{c_f^2} - 1}}, \quad (5)$$

while for the rod we have

$$\alpha_{\text{rod}} \approx \frac{\pi \rho a^2 \omega^3 \nu^2}{2 c_o^3}. \quad (6)$$

Combining Eqs. (2) through (6) we arrive at the low-frequency asymptotic expressions for the above defined quality factor in the case of the plate and the rod. For the plate, we obtain

$$Q_{\text{plate}} \approx \frac{\rho d \omega c_f^2}{2 c_o^3} \sqrt{\frac{c_o^2}{c_f^2} - 1}. \quad (7)$$

Similarly, for the rod we can write the quality factor as follows:

$$Q_{\text{rod}} \approx \frac{2}{\pi} \ln \left(\frac{a \omega}{c_o} \sqrt{\frac{c_o^2}{c_f^2} - 1} \right). \quad (8)$$

To design a successful experiment, we need to maximize the magnitude of the quality factor (it is positive for the plate and negative for the rod). Several parameters can be changed to achieve this goal, including the characteristic dimension and velocity of the specimen, the frequency, and the density and velocity ratios between the solid and the fluid. Among all these parameters, frequency seems to be the one that can be most easily varied. Sweeping the frequency over a carefully chosen and sufficiently wide frequency range will be shown to achieve the desired goal in the case of the plate but has serious limitations in the case of the rod. To help illustrate the difference between the plate and the rod we refer to Fig. 2, which shows a schematic diagram of the dispersion curves for the fundamental dilatational modes in free and fluid-loaded plates and rods. The solid lines represent the phase velocity of the modes in the free case, and the dotted

lines give the velocity in the fluid-loaded case. In this schematic figure, the fluid loading-induced changes in the velocity of this mode have been greatly exaggerated for clarity. It is very well known that at high frequencies the velocity of the fundamental dilatational modes approach the velocity of the Rayleigh wave in both the free plate and the rod. Similarly, in the fluid-loaded case, those two modes approach the leaky Rayleigh mode which has a slightly higher velocity than the true Rayleigh mode (approximately 0.5% higher for water-loaded aluminum). As shown in Fig. 2, in the case of the plate, the velocity of the fundamental dilatational mode increases due to fluid loading throughout the entire frequency range, whereas in the case of the rod, the velocity first drops at low frequencies due to fluid loading and then switches over to be higher than the velocity in the free case. For typical Poisson ratios, the switch between negative and positive radiation-loading effects occurs at around $a\omega/c_s \approx 1.3$. Our measurements in the case of the rod should be carried out at frequencies well below this switch-over frequency. As a matter of fact, the negative radiation-loading effect peaks at around $a\omega/c_s \approx 0.6$. More importantly, the magnitude of the above-defined quality factor, i.e., the ratio between the radiation-induced velocity and attenuation effects, becomes higher and higher as the frequency is decreased. In a plate, the measurements should be carried out at relatively high normalized frequencies ($d\omega/c_s$) over a short propagation distance in order to avoid excessive attenuation upon immersion. In contrast, in a rod, the corresponding measurements should be carried out at relatively low normalized frequencies ($a\omega/c_s$) over a long propagation distance to assure significant phase variation upon immersion. This trend was expected, as it has been shown previously that the radiation loading in inviscid fluids is a first-order effect in the case of a rod, whereas it is a second-order effect for a plate.¹⁷ At first sight, this result could suggest that the radiation-induced velocity change is easier to observe in the rod than it is in the plate. This fact is true in a strictly inviscid fluid up to the point where the negative effect of fluid loading on the velocity of the fundamental dilatational mode in the rod reaches its maximum. However, this point is reached at a very low frequency. Because the effect in the case of the plate can be maintained, and actually increased, to much higher frequencies, it is possible to measure a much bigger effect in the plate by going to higher frequencies, where the otherwise inevitable viscosity effects become negligible. Theoretically, the quality factor can also be increased in the case of the rod by lowering the frequency, but the logarithmic nature of the increase makes this approach very ineffective. More importantly, as we shall show in the next section, lowering the frequency increases the otherwise negligible effects of the inevitable viscous drag between the solid and the fluid, thereby rendering the detection of radiation-induced velocity change less feasible.

B. Viscous drag

The most important extraneous effect that can interfere with the experimental verification of the previously described opposite effect of radiation loading on the velocity of the fundamental dilatational modes in immersed thin plates

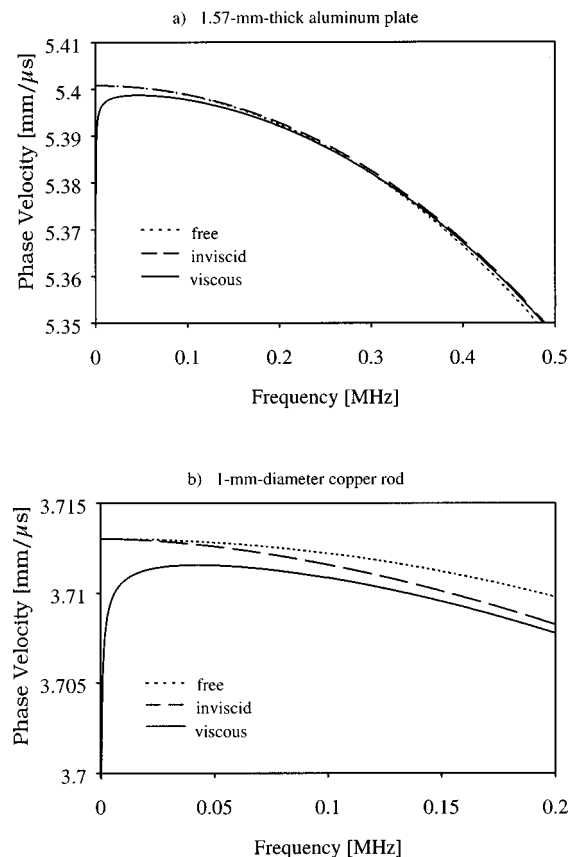


FIG. 3. Dispersion curves in an aluminum plate (a) and copper rod (b) immersed in water.

and rods is the inherent viscous drag presented by real fluids. Because the surface vibration of these modes at low frequencies is almost entirely tangential, even in low-viscosity fluids like water, fluid loading is dominated by viscous effects below a certain frequency. The viscosity-induced velocity change in thin plates and rods can be easily calculated by modeling the viscous fluid as a hypothetical isotropic solid having a frequency-dependent rigidity of $-i\omega\eta$, where η denotes the viscosity of the fluid.^{18,19} In this way, the vorticity mode associated with the viscosity of the fluid is formally described as the shear mode in the fictitious solid. Figure 3 shows the dispersion curves in an aluminum plate (a) and copper rod (b) immersed in ideal inviscid and ordinary viscous water along with the corresponding dispersion curves in the free specimens. The density, dilatational velocity, shear velocity, and thickness of the aluminum plate were taken as $\rho_{sa} = 2700 \text{ kg/m}^3$, $c_{da} = 6323 \text{ m/s}$, $c_{sa} = 3100 \text{ m/s}$, and $d = 1.57 \text{ mm}$, respectively, to model the actual specimen used in our experiments. Similarly, the density, dilatational velocity, shear velocity, and radius of the copper rod were taken as $\rho_{sc} = 8900 \text{ kg/m}^3$, $c_{dc} = 4700 \text{ m/s}$, $c_{sc} = 2260 \text{ m/s}$, and $a = 0.5 \text{ mm}$, respectively. For water, the density was taken as $\rho_f = 1000 \text{ kg/m}^3$, the sound velocity as $c_f = 1000 \text{ m/s}$, and the viscosity as $\eta = 10^{-3} \text{ kg/ms}$.

The theoretically predicted effect of the viscosity of the fluid on the velocity of the fundamental dilatational wave in a thin plate is illustrated in Fig. 3(a). The dotted line shows the dispersion curve of this mode in the fluid-free plate; the dashed line shows the velocity of the same mode as a func-

tion of frequency in the plate as it is immersed in ideal inviscid water to represent the separate effect of radiation loading, whereas the solid line shows the dispersion curve for this mode as the plate is immersed in ordinary viscous water to represent the total fluid loading. Below 300 kHz, fluid loading actually decreases the phase velocity in the plate as a result of viscous drag. However, at higher frequencies, fluid loading is dominated by the radiation effect and the velocity increases with respect to the free plate. The effect of fluid loading on the velocity of the fundamental dilatational wave in a thin rod is illustrated in Fig. 3(b). Again, the dotted line shows the dispersion curve of this mode in the fluid-free specimen, and the dashed line shows the velocity of the same mode as a function of frequency in the rod as it is immersed in ideal inviscid water to represent the separate effect of radiation loading, whereas the solid line shows the dispersion curve for this mode as the rod is immersed in ordinary viscous water to represent the total fluid loading. We notice that as the copper wire is loaded by inviscid water, the velocity of the dilatational wave decreases. This reduction in the velocity increases as the frequency is increased. When the wire is immersed in viscous water, there is an additional drop in the velocity of the dilatational mode caused by the viscosity of the fluid. As the frequency increases, the effect of viscosity diminishes and the effect of leakage into the fluid becomes dominant.

Since our main interest is to measure the radiation part of fluid loading on this mode of wave propagation, and since the always-negative viscosity effect dominates at low frequencies while the radiation effect diminishes, it is necessary to limit our experimental measurements to a minimum frequency where the viscosity effect becomes negligible. However, the highest possible frequency in the case of the rod was limited by the fact that the reduction in the velocity of this mode is a low-frequency effect, and it reverses sign at relatively modest frequencies after which the velocity of this mode in the rod actually increases. In the case of the plate, such maximum does not exist as radiation loading always increases the phase velocity of the fundamental dilatational mode, regardless of the frequency. It should be mentioned, however, that once the dispersion becomes excessive we can no longer use the approximations given in Eqs. (5) and (7). In conclusion, viscosity will adversely affect our ability to measure separately the radiation component of the fluid-loading induced velocity drop in thin rods, while the leaky attenuation limits our measurements to low frequencies. In comparison, viscosity will have negligible effect on the measurement of the radiation component of the fluid-loading induced velocity increase in thin plates, when the leaky attenuation limits our measurements to high frequencies anyway. The experimental results and the appropriate frequency ranges for the two cases will be presented and fully discussed in the following section.

II. EXPERIMENTAL METHOD

The experimental setup used in measuring the relative change in the velocity of the fundamental dilatational waves in thin plates and rods due to fluid loading is shown in Fig. 4. Both transmission and reception were accomplished by sym-

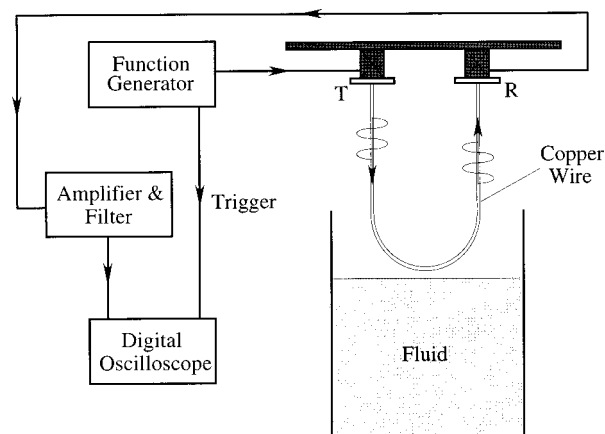


FIG. 4. The experimental setup used in investigating the fluid-loading effect on the dilatational wave velocity in thin rods and plates.

metrically mounted longitudinal transducers to reject all flexural modes. However, the much lower group velocity of these modes in thin plates and rods made it possible to further eliminate spurious flexural vibrations by simply gating out the faster fundamental longitudinal mode. The arrangement shown in Fig. 4 is for the case of a thin rod and is also used for the thin plate with minor modifications. The negative effect of fluid loading on the velocity of the lowest-order dilatation mode in a thin rod can be observed only in a narrow window limited from below by viscosity effects and also from above by a turning point beyond which the effect changes sign. Therefore, we had to use a very long and very thin wire which was bent as shown in Fig. 4. The leaky wave produced by the wire is diverging and will not interfere with the guided wave propagating in the wire even if it hits part of it. We made sure that the radius of curvature of the bent wire is high enough not to cause any perceivable attenuation or dispersion at the inspection frequencies used. In contrast, the positive effect of fluid loading on the velocity of the lowest-order dilatation mode in a thin plate can be observed in a wide frequency range anywhere above a lower limit set by viscosity effects. Therefore, we could use a relatively short plate without bending it like the wire shown in Fig. 4 (the sealed receiver was underwater). One additional difference between the wire and plate was the difference in material. A longitudinal contact transducer is very difficult to mount on the end of a thin rod, therefore, a copper wire was used which could first be soldered to a very thin copper foil which in turn was glued to the face of the transducer as described in Ref. 16. In the case of the plate, coupling is much easier, but the necessarily higher inspection frequency requires a less attenuating medium; therefore, an aluminum plate was used (at three times higher frequency, the grain-scattering induced attenuation in copper would be almost 100 times higher). However, it should be emphasized that our main goal is to experimentally demonstrate the opposite sign of the fluid-loading effect on rods and plates; therefore, the difference in material properties is of no particular importance. It should also be mentioned that the large dimensions of the water tank combined with the relatively short excitation and propagation times of the tone burst signals allowed us to completely disregard the acoustic modes generated in the fluid itself.

The basic arrangement uses two ultrasonic contact transducers in a pitch–catch mode of operation. One of the transducers acts as a transmitter and the other as a receiver. The transmitter is driven by a tone burst excitation generated by DS345 Stanford Research Systems synthesized function generator at the desired frequency. The number of cycles in the tone burst is adjusted according to the frequency, where a small number of cycles ($\approx 3\text{--}5$) is used at low frequencies, and a large number of cycles ($\approx 10\text{--}30$) is used in the high-frequency range. The transmitted signal is detected by the receiver and passed through an amplifier and bandpass filter, then displayed using a LeCroy 9310 digital oscilloscope. The function generator is also used to trigger the oscilloscope. The specific experimental arrangement and procedure for the plate and the rod are detailed later.

It should be mentioned that the velocity of the dilatational wave in thin plates and rods perceptibly decreases with increasing temperature. This extraneous effect was found to be more serious in thin rods which have low thermal capacity against temperature variations and require more accurate measurements at the same time. For example, the velocity of the fundamental dilatational mode in a copper wire decreased in a linear fashion with the temperature of the water bath at a rate of approximately $6 \times 10^{-4}/^\circ\text{C}$. With relatively simple means, the water temperature can be maintained at the ambient air temperature within better than $\pm 1^\circ\text{C}$, which effectively eliminates any gross temperature variation in the specimen as it is immersed in the water bath. However, an additional variation occurs when the specimen is subsequently taken out of the bath due to evaporation of the water from the surface, which is much more difficult to control. This evaporation causes the structure to cool temporarily, thereby increasing the velocity of the dilatational wave. Fortunately, this effect can be kept under control by waiting long enough for the temperature to stabilize as indicated by the received ultrasonic signal's returning to its original state recorded before immersion.

A. Thin plate

A $1.57 \times 267 \times 127\text{-mm}$ aluminum plate was used in this experiment (the density and dilatational and shear velocities of the specimen were given in the previous section). Two 1.0-MHz, 0.5-in.-diameter transducers were mounted on the two opposite edges normal to the longer sides of the plate and operated in pitch–catch mode. This arrangement proved to be better than the pulse–echo configuration as it reduced adverse cross talk, better separated the multiple reflections, and minimized the spurious reflections from the edges of the plate within the time window of interest. Figure 5(a) shows typical rf signals recorded at a carrier frequency of 950 kHz with a 200-kHz filter bandwidth. To reduce the effects of wave interaction with the surface of the water bath, approximately 5 mm of the plate is immersed in water even in the “free” state [top signal in Fig. 5(a)]. In the “immersed” state [bottom signal in Fig. 5(a)] the immersed part is increased to a larger value, e.g., 25 mm, i.e., by $l = 20\text{ mm}$, using a computer-controlled translation stage. The actual distance l was varied at different frequencies based on the attenuation of the signal. Figure 5(b) shows the correspond-

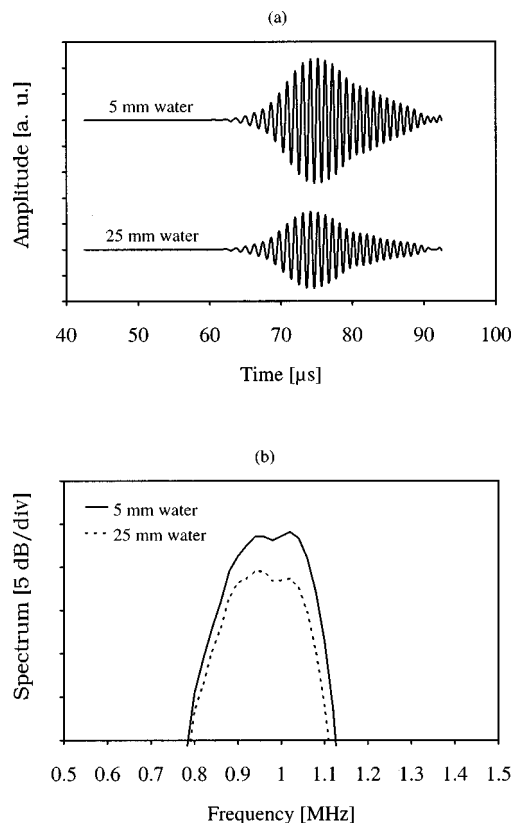


FIG. 5. Typical (a) rf signals in a 1.57-mm-thick aluminum plate immersed in water at 950 kHz, and (b) the corresponding spectrum.

ing spectra of the signals displayed in Fig. 5(a). An average fluid-loading induced attenuation of approximately $\Delta L \approx 5\text{ dB}$ was maintained during these measurements by varying the distance l from 30 mm at low frequencies to 10 mm at high frequencies.

It is possible to estimate the fluid-loading induced velocity change by simply monitoring a given zero crossing in the detected rf signal. However, careful inspection of the recorded rf signals revealed that some of the zero crossings exhibited higher or lower than average shift as a result of perceptible dispersion in spite of the relatively narrow bandwidth. This remnant dispersion, which is mainly caused by inevitable spurious standing waves interfering with the propagating wave, produces a localized phase variation that can be eliminated by averaging over the whole length of the signal. Therefore, the relative change in the velocity of the dilatational wave in the plate was determined by measuring the phase of the total rf signal recorded in the free and fluid-loaded states at the desired frequency. Calculating the phase of the total digitized rf signal by FFT for the free and the fluid-loaded cases gives a more realistic estimate of the change in the velocity than localized zero-crossing measurements. The phase difference $\Delta\Phi$ between the loaded and unloaded spectra was then converted to the sought relative velocity change $\Delta c/c_o$ via Eq. (1).

B. Thin rod

A 1.0-mm-diameter, 500-mm-long copper wire was used in the measurements (the density and dilatational and

shear velocities of the specimen were given in the previous section). Two 2.25-MHz, 0.25-in.-diameter transducers were attached to the copper wire through thin copper foils that were soldered to the wire to ensure stable and good mechanical coupling. The two transducers were glued on the other side to an aluminum plate through two rubber cushions to create a stable structure that can be easily handled during the experiment. The two rubber cushions acted as vibration isolators that were necessary to isolate the transducers from the elastic aluminum plate, thereby minimizing any structural vibrations that can couple from the transmitter to the receiver through the plate. The copper wire was bent into a “U” shape and approximately 95% of its length was loaded by water.

The relative velocity change in this case was determined by monitoring a zero crossing in the rf signal with and without fluid loading. Unlike the case of the plate, all the zero crossings in this case seemed to be moving in the same direction with approximately the same amount indicating the absence of strong standing-wave interference patterns within the structure. The time shift of the monitored zero crossing was recorded and converted to relative velocity change using the following simple equation:

$$\frac{\Delta c}{c_o} \approx \frac{\Delta t c_o}{/}, \quad (9)$$

where Δt is the time delay of the zero crossing, $/$ is the portion of the length of the wire that is loaded by water, and c_o is the low-frequency asymptotic value of the phase velocity in the free rod. Due to the finite length of the wire, and possibly to the finite curvature of the bent specimen, a weak standing-wave pattern is established in the wire as the longitudinal wave propagates from the transmitter side to the receiver side and reflects back to the transmitter side. This weak standing-wave pattern was reduced by averaging the results obtained at more than one zero crossing in the rf signal.

III. EXPERIMENTAL RESULTS AND DISCUSSION

In this section, we present our experimental results on the effect of fluid loading on the velocity of the fundamental dilatational modes in thin plates and rods and compare them with the theoretical predictions obtained by numerically solving the corresponding exact dispersion equations for the cases of inviscid and viscous fluid loading. We will start by presenting the results for the plate and follow that with the results for the rod.

A. Thin plate

Figure 6 shows the relative change in the velocity of the fundamental dilatational mode in a 1.57-mm-thick aluminum plate immersed in water as a function of frequency. The change in the velocity is normalized to the low-frequency asymptotic value ($c_o = 5400$ m/s) of the velocity of the dilatational mode in the free plate. The solid circles represent the average of five experimental measurements of the relative change in the velocity at the same excitation frequency, and the bars superimposed on them represent the estimated error

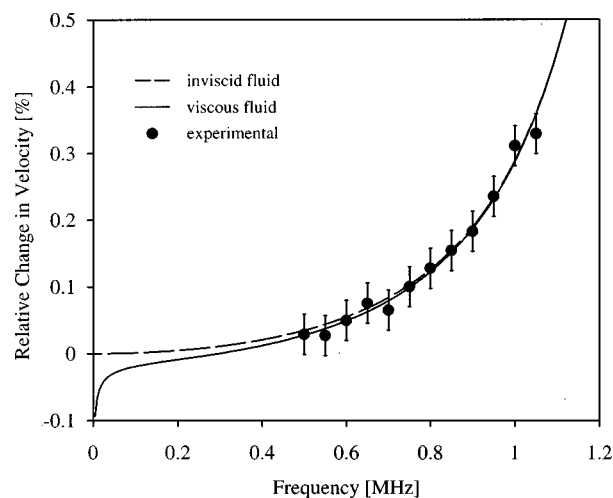


FIG. 6. Relative change in the velocity of the dilatational wave in a 1.57-mm-thick aluminum plate immersed in water.

in the measurements ($\approx \pm 0.03\%$), which was based on the largest scatter in the measured data. The dashed line represents the theoretically predicted relative change in the velocity of this mode for the plate immersed in ideal inviscid water. This includes only the effect due to energy leakage into the fluid. The solid line shows the results for the case of viscous water which includes both the component due to leakage into the fluid and the component due to viscosity which is relatively weaker at high frequencies where the measurements were made. The frequency range in which the experimental measurements were performed extends from 520 kHz to 1.0 MHz. Excessive attenuation and dispersion of this mode restricted the highest frequency at which we could measure this effect. The lowest frequency at which the measurements were carried out was limited mainly by the bandwidth of the ultrasonic transducers, by the viscosity effect, which becomes more dominant below about 300 kHz, and by the reflections from the side edges of the plate which significantly distorted our signal below 500 kHz.

We can conclude from Fig. 6 that there is an excellent quantitative agreement between the experimentally measured relative change in the velocity of the fundamental dilatational mode in a thin plate and the corresponding predictions from theoretical models. The experimental results clearly verify that radiation loading indeed increases the velocity of the fundamental dilatational mode in a thin plate.

B. Thin rod

Figure 7 shows the relative change in the velocity of the fundamental dilatational wave in a 1.0-mm-diameter copper rod immersed in water. The change in the velocity is normalized to the low-frequency asymptotic value ($c_o = 3710$ m/s) of the velocity of the dilatational mode in the free rod. The solid circles again represent the average of five experimental measurements of the relative change in the velocity at the same excitation frequency, and the bars superimposed on them represent the estimated error in the measurements ($\approx \pm 0.015\%$), which was again based on the largest scatter in the measured data. In this case, due to the lack of strong interference patterns that could distort the detected signal,

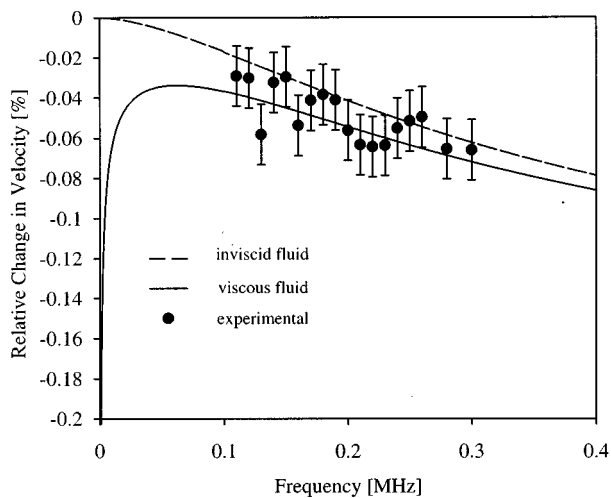


FIG. 7. Relative change in the velocity of the dilatational wave in a 1.0-mm-diameter copper wire immersed in water.

the simpler time-delay measurement of a zero crossing in the rf signal was used to determine the relative velocity change. The solid line in the figure represents the theoretical prediction in the case of a thin rod immersed in ordinary viscous water, and the dashed line represents the case of idealized inviscid water. The frequency range over which the experimental measurements were taken extends from 100 to 300 kHz. The highest measuring frequency in this case was limited by the fact that the negative radiation-loading effect in the case of the rod occurs only up to a certain limit, beyond which this effect becomes positive, and that even below this switch-over frequency the fluid-loading induced attenuation becomes excessively high for precise velocity measurements. This behavior was previously illustrated in Fig. 2 and discussed in a previous section. This measurement is particularly difficult since the measuring frequency range is also limited from below at about 100 kHz by inevitable viscosity effects which exert a much stronger influence on thin rods than plates. Although the radiation loading-induced velocity change in a thin rod is a first-order effect, its actual value is very small, which makes it extremely difficult to produce good agreement with theoretical predictions.

Considering the experimental difficulties associated with this measurement, the agreement between the experimentally obtained results and the theoretical predictions for the relative change in the velocity of the fundamental dilatational wave in a thin rod is good. These experimental results verify that radiation loading indeed decreases the velocity of the fundamental dilatational wave in a thin rod.

IV. CONCLUSIONS

In spite of the basic similarities between the fundamental dilatational wave modes in thin plates and rods, radiation loading exerts an opposite effect on their respective phase velocities. It was shown recently that this somewhat unexpected opposite effect is caused by the different nature of radiation loading in the two cases, as the radiation impedance exerted by the fluid on thin plates is purely real while it is dominantly imaginary in the case of thin rods.¹⁷ In this paper, our main goal was to experimentally verify these ear-

lier analytical predictions. The relative change in the velocity of the dilatational mode in thin plates and rods was measured over a substantially wide frequency range. Simple theoretical considerations showed that, in the case of the rod, the frequency range over which the negative radiation-loading effect can be unequivocally observed is very narrow. The problem is less serious in the case of the plate, as the positive radiation-loading effect is maintained to much higher frequencies where viscosity effects are negligible. Because of these limitations on the frequency range over which the measurements could be carried out, the agreement between the experimental results and the theoretical predictions was better in the case of the plate as compared to the rod. Although the accuracy of the measurements in the case of the rod ($\approx \pm 0.015\%$) is slightly better than that in the case of the plate ($\approx \pm 0.03\%$), the absolute value of the detectable velocity change is much less for the rod than for the plate; therefore, positive verification of the predicted velocity change is more difficult in the case of the rod.

Considering the fact that the velocity effect is much smaller than the accompanying attenuation effect, and that its relative value is only on the order of a fraction of a percent, we can only conclude that such measurements are very demanding and require the minimization of all other effects that can mask the radiation-loading effect. Such effects mainly include the effect of viscosity of the fluid and to some degree, temperature variations. In our measurements, both of these effects were minimized to a level where the radiation-loading effect was dominant. Generally, the agreement between the experimental results and the theoretical predictions in both cases of thin plates and rods was good. The experiments unequivocally verified that radiation loading decreases the velocity of the dilatational wave in thin rods and increases it in thin plates. To our knowledge, these measurements constitute the first experimental verification of the previously reported opposite effect that radiation loading exerts on the velocity of the fundamental dilatational modes in immersed thin plates and rods.

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